

An Integrated Approach to Large Amplitude Internal Wave Dynamics and Their Surface Signatures

Wooyoung Choi
Department of Mathematical Sciences
New Jersey Institute of Technology
University Heights
Newark, NJ 07102-1982
phone: (973) 642-7979 fax: (973) 596-5591 email: wychoi@njit.edu

Award Number: N00014-08-1-0377

LONG-TERM GOALS

The long-term goal of this research is to develop an integrated model to describe the generation, propagation, and transformation of large amplitude internal solitary waves through interaction with bottom topography and to investigate sea surface roughness associated with the internal wave motions.

OBJECTIVES

The objective of this project is to investigate large amplitude internal wave dynamics and the associated surface roughness changes with developing an integrated internal wave prediction/remote sensing model based on (1) the strongly nonlinear internal wave model initialized with in-situ data and (2) the fully nonlinear surface wave model coupled with the surface current predictions of the internal wave model.

APPROACH

Our approach is to solve numerically the strongly nonlinear internal wave model for a two-layer system (Choi & Camassa, 1999) derived via an asymptotic expansion method under the sole assumption that the waves are long compared to the total water depth with including the leading order non-hydrostatic corrections. Once the surface current field induced by internal solitary waves is obtained from the numerical solution of the strongly nonlinear internal wave model, the evolution of surface gravity-capillary waves is studied using a surface wave prediction model that solves a closed system of nonlinear evolution equations (West *et al.*, 1987; Choi, 1995) for the free surface elevation and the velocity potential evaluated at the free surface.

WORK COMPLETED

- (1) The strongly nonlinear internal wave model suffering from the Kelvin-Helmholtz instability due to a velocity jump induced by the interfacial displacement is regularized by changing the dispersive characteristics of short waves that the original model poorly describes.
- (2) Comparison between the strongly and weakly nonlinear internal long wave models is made for an internal solitary wave propagating over bottom topography and the effects of strong nonlinearity on both the interfacial displacement and the induced horizontal current at the free surface are examined.

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE 30 SEP 2008		2. REPORT TYPE Annual		3. DATES COVERED 00-00-2008 to 00-00-2008	
4. TITLE AND SUBTITLE An Integrated Approach To Large Amplitude Internal Wave Dynamics And Their Surface Signatures				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) New Jersey Institute of Technology, Department of Mathematical Sciences, University Heights, Newark, NJ, 07102-1982				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES code 1 only					
14. ABSTRACT The long-term goal of this research is to develop an integrated model to describe the generation, propagation, and transformation of large amplitude internal solitary waves through interaction with bottom topography and to investigate sea surface roughness associated with the internal wave motions.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Same as Report (SAR)	18. NUMBER OF PAGES 5	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

- (3) Large amplitude internal solitary wave solutions are found for a two-layer system where each layer has a constant buoyancy frequency and are compared with those for a system of two constant density layers.
- (4) Direct numerical simulations of surface gravity waves interacting with the surface current induced by an internal solitary wave are conducted to investigate the generation of short waves crucial for microwave radar backscatter.

RESULTS

1. Regularization of the strongly nonlinear internal wave model

With the rigid lid assumption for the top boundary, the strongly nonlinear model can be written as

$$\eta_{it} + (\eta_i u_i)_x = 0, \quad \eta_1 = h_1 - \zeta, \quad \eta_2 = h_2 + \zeta$$

$$u_{it} + u_i u_{i,x} + g\zeta_x = -\frac{P_{i,x}}{\rho_i} + \frac{1}{\eta_i} \left(\frac{1}{3} \eta_i^3 G_i \right)_x, \quad G_i = u_{i,xt} + u_i u_{i,xx} - u_{i,x}^2$$

where $\eta_i(x,t)$ is the local thickness of the i -th layer, h_i is the unperturbed thickness of the i -th layer, and $u_i(x,t)$ is the layer-mean horizontal velocity. When this model is solved numerically, it is shown in Jo & Choi (2002) that this model suffers from the Kelvin-Helmholtz instability since u_1 is different from u_2 . To suppress such instability, the model is re-written in terms of $\hat{u}_i(x,t)$, the horizontal velocity evaluated at the top and bottom boundaries for $i=1$ and 2, respectively, as

$$\eta_{it} + \left[\eta_i \left(\hat{u}_i - \frac{1}{6} \eta_i^2 \hat{u}_{i,xx} \right) \frac{1}{6} \right]_x = 0$$

$$\hat{u}_{i,t} + \hat{u}_i \hat{u}_{i,x} + g\zeta_x = -\frac{P_{i,x}}{\rho_i} + \left[\frac{1}{2} \eta_i^2 \left(\hat{u}_{i,xt} + \hat{u}_i \hat{u}_{i,xx} - \hat{u}_{i,x}^2 \right) \right]$$

and it is found that the solitary wave solution of the original model is stable with the regularized model, as shown in figure 1.

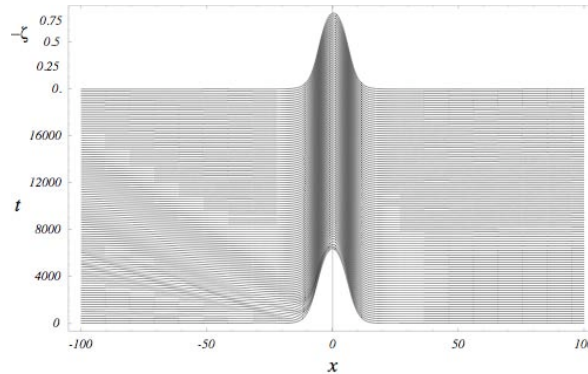


Figure 1. Numerical solution of the regularized model for the evolution of a single internal solitary wave of wave amplitude of $0.8h_1$ in a frame of reference moving with the solitary wave. The initial condition is not the exact solution of the regularized model, but no sign of the Kelvin-Helmholtz instability is observed.

2. Comparison between strongly and weakly nonlinear internal wave models

The comparison between the strongly and weakly nonlinear internal wave models for a single internal solitary wave propagating over bottom topography are conducted and the results are shown in figure 2. Compared with the strongly nonlinear model, the weakly nonlinear model overpredicts the displacement of the interface and considerable differences in phase are observed.

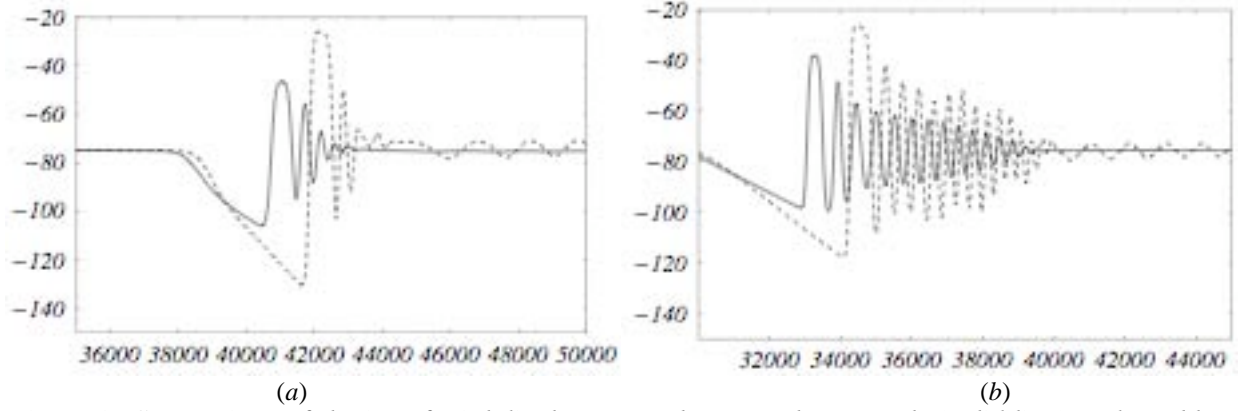


Figure 2. Comparison of the interfacial displacements between the strongly (solid line) and weakly (dashed line) nonlinear internal wave models. (a) $t=41942$; (b) $t=51920$.

3. Large amplitude internal solitary waves in a linearly-stratified fluid

By adopting an asymptotic expansion technique similar to that of Choi & Camassa (1999) for a system of two constant density layers, approximate solitary wave solutions of the DJL equation for a two-layer system of constant buoyancy frequencies (piecewise linear density profiles) are computed using a dynamical system approach. A result is shown in figure 3 and a detailed description of such solutions for various physical parameters can be found in Goullet & Choi (2008).

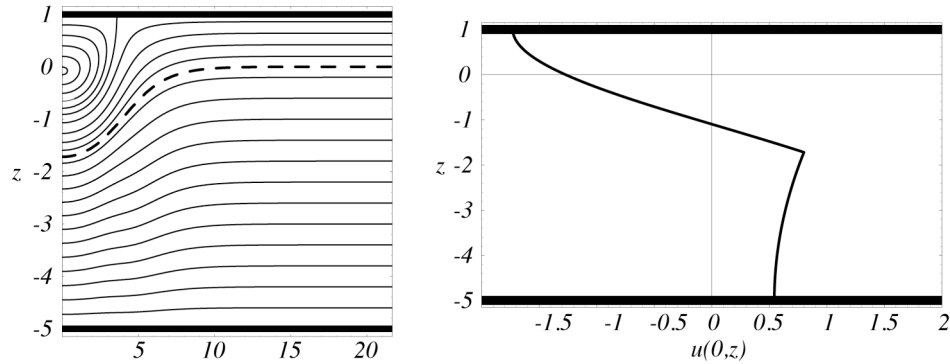


Figure 3. Streamlines and the horizontal velocity at the center for mode-1 internal solitary wave of wave speed $c/(gh_1)^{1/2}=1.2$. The upper layer is linearly stratified while the density of the lower layer is constant. The density across the interface is continuous and the depth ratio of the thickness of the lower layer to that of the upper layer is 5.

4. Evolution of surface gravity waves interacting with the surface current induced by a single internal solitary wave

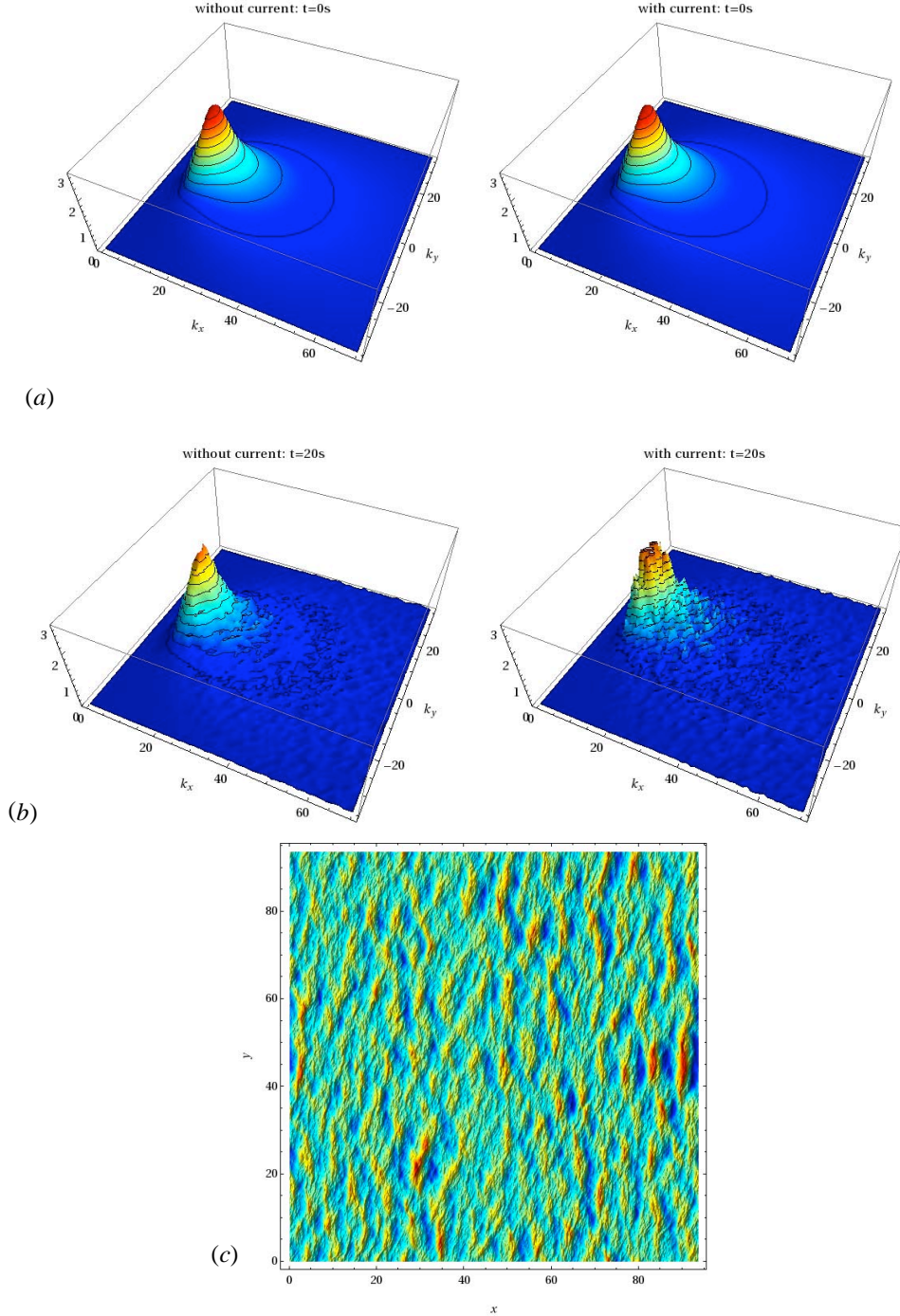


Figure 4. Nonlinear interaction between a surface wave field characterized by the JONSWAP spectrum of peak enhancement factor $\gamma=3.3$, significant wave height $H_s=0.1$ m, and peak wave period $T_p=2$ sec, and a surface current generated by an internal solitary wave of amplitude $a=1.25$ m in a two layer system of $h_1=2.5$ m and $h_2=7.5$ m. (a) Initial wavenumber spectrum; (b) Spectrum at $t=20s$; (c) Surface wave field at $t=20$ with the surface current. The generation of high wavenumber components can be seen clearly from the wave number spectrum at $t=20$ with the current as shown in (b).

IMPACT/APPLICATIONS

The project is expected to provide a comprehensive tool for predicting and monitoring the strongly nonlinear internal wave activity and surface signatures.

RELATED PROJECTS

This investigation is closely related to other projects funded by the ONR Non-Linear Internal Wave Initiative (NLIWI). Comparison of our numerical solutions with field data for the propagation of internal solitary waves and SAR images is under progress.

The PI of this project is working on the University of Michigan MURI project entitled “Optimum Vessel Performance in Evolving Nonlinear Wave Fields” and investigating the energy transfer and dissipation of short waves due to wave breaking whose understanding is important to radar remote sensing of internal solitary waves is being investigated.

REFERENCES

Choi, W. 1995 Nonlinear evolution equations for two-dimensional surface waves in a fluid of finite depth, *J. Fluid Mech.*, **295**, 381-394.

Choi, W. & Camassa, R. 1999 Fully nonlinear internal waves in a two-fluid system. *J. Fluid Mech.* 1-36.

West, B. J., Brueckner, K. A., Janda, R. S., Milder, D. M. and R. L. Milton, 1987 A new numerical method for Surface Hydrodynamics, *J. Geophys. Res.*, **92**, 11803-11824.

PUBLICATIONS

Goullet, A. and Choi, W. Large amplitude solitary waves of a linearly stratified flow. *Phys. of Fluids*, 2008, **20**, 2008, 066604 [published, refereed]

Jo, T. and Choi, W. On stabilizing the strongly nonlinear internal wave model, *Studies in Applied Math*, **120**, 2008, 65-85 [published, refereed]

Barros, R. and Choi, W. On the hyperbolicity of two-layer flows. Proceedings of the FACM Conference, 2008, submitted.